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Superconvergence for finite element approximation of a convection-diffusion equation using graded meshes

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ABSTRACT

Given the model convection-diffusion problem

$$\begin{aligned} -\varepsilon\Delta u + \vec{b}\cdot\nabla u + cu = & \quad f, \quad \text{in} \ \ \Omega, \\ u = & \quad 0, \quad \text{on} \ \ \partial\Omega, \end{aligned}$$

we analyze the approximation by standard bilinear finite elements using the graded meshes introduced in [1].

Our main goal is to prove superconvergence results of the type known for standard elliptic problems, namely, that the difference between the finite element solution and the Lagrange interpolation of the exact solution, in the ε-weighted H^1-norm, is of higher order than the error itself. The constant in our estimate depends only weakly on the singular perturbation parameter. Our work is based on the techniques of [3] and [2].

As a consequence of the superconvergence result, we obtain optimal order error estimates in the L^2-norm. Also, we show how to obtain a higher order approximation by a local postprocessing of the computed solution.

References

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