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Galerkin Finite Element Approximations of Elliptic Problems and Their Hybridization

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ABSTRACT

The first hybridization of a finite element method was proposed in 1965 by Fraejis de Veubeke as an implementation of numerical methods for solving the equations of linear elasticity. In 1985 Arnold and Brezzi showed that the hybridization is more than an implementation trick. They proved that the new unknown, which plays a role of a Lagrange multiplier for restoring the interelement continuity of the normal derivative across the interfaces, contains *extra* information about the exact solution. After another two decades, a new perspective on hybridization emerged [2] with the characterization of the approximate trace as the solution of certain interface problem.

In the last decade, especially after the fundamental paper of Arnold, Brezzi, Cockburn, and Marini, [1], the discontinuous Galerkin method (DG FEM) has emerged as a powerful technique for solving PDE's that features flexibility in meshing and polynomial spaces, easily controlled stability, local conservation properties, and possibility to glue mixed, standard Galerkin, nonconforming, and other finite element approximations. However, DG FEM uses excessive number of degrees of freedom. Hybridization is one possible way to substantially reduce the number of degrees of freedom. The main ideas of the hybridization combined with the technique of lifting operators from discontinuous Galerkin (DG) approximations led to a unified hybridization technique [3] for DG, mixed, nonconforming, and conforming finite element approximations of second order elliptic problems.

In the talk we shall discuss this general hybridization framework for second order elliptic problems, which is characterized by: (1) the finite element spaces of the local solutions, (2) the numerical traces of the solution and the flux, and (3) the space of the Lagrange multiplier. We first show, that this framework reproduces the known hybridization of the standard mixed finite element approximations, in particular those of [2]. Next we study the hybridization of the interior penalty (IP) DG method and show that the proposed procedure fully characterizes the class of hybridizable IP DG schemes. We demonstrate that from the known IP DG methods only the IP scheme due to Ewing, Wang, and Yang, [4], is hybridizable. Finally, we show that most of the LDG schemes are hybridizable and fully characterize them.

The talk is based on the joint work [3] of the author with Cockburn and Gopalakrishnan.

References

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