

## The Mathematical Secret of Flight

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### ABSTRACT

We show by computational solution of the incompressible Navier-Stokes equations with friction force boundary conditions, that the classical inviscid circulation theory by Kutta-Zhukovsky for lift of a wing and laminar viscous boundary layer theory by Prandtl for drag, which have dominated 20th century flight mechanics, do not correctly describe the real turbulent airflow around a wing. We show [1, 5] that lift and drag essentially originate from a turbulent wake of counter-rotating rolls of low-pressure streamwise vorticity generated by a certain instability mechanism of potential flow at rear separation. We use a stabilized finite element method with duality-based a posteriori error control, which allows computational prediction of the flight characteristics of an airplane using millions of meshpoints without resolving thin boundary layers, instead of the impossible quadrillions required according to state-of-the-art for boundary layer resolution.

We present applications to the flight of an airplane, helicopter, paraglider, bird, boomerang, frisbee, discus, and to sailing, swimming and paddling, as well as spin in ball sports, and the action of a propeller and wind turbin, [4, 1]. The new theory of lift and drag is a consequence of the recently published resolution of d'Alembert's paradox [2, 3] and a new theory for separation in slightly viscous flow [6].

### References

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