

Finite element approximation of the convection-diffusion equation: subgrid-scale spaces, local instabilities and anisotropic space-time discretizations

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ABSTRACT

The objective of this talk is to give an overview of the finite element approximation of the convection-diffusion equation that we have been developing in our group during the last years, together with some recent methods. We discuss three main aspects, namely, the global stabilization in the convective dominated regime, the treatment of the local instabilities that still remain close to layers when a stabilized formulation is used and the way to deal with transient problems.

The starting point of our formulation is the variational multiscale framework. The main idea is to split the unknown into a finite element component and a remainder that is assumed that the finite element mesh cannot resolve. A closed form expression is then proposed for this remainder, referred to as subgrid-scale. When inserted into the equation for the finite element component, a method with enhanced stability properties is obtained. In our approach, we take the space for the subgrid-scales orthogonal to the finite element space.

Once global instabilities have been overcome, there are still local oscillations near layers due to the lack of monotonicity of the method. Shock capturing techniques are often employed to deal with them. Here, our point of view is that this lack of monotonicity is inherent to the integral as duality pairing intrinsic to the variational formulation of the problem. We show that if appropriate weighting functions are introduced when computing the integral, giving a reduced weight to layers, the numerical behavior of the method is greatly improved.

The final point we treat is the time integration in time-dependent problems. Most stabilized finite element methods require a link between the time step size of classical finite difference schemes in time and the mesh size employed for the spatial discretization. We show that this can be avoided by considering the subgrid-scales as time dependent, and discretizing them in time as well. That allows us to perform a complete numerical analysis which is not restricted by any condition on the time step size, thus permitting anisotropic space-time discretizations

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