

Finite element approximation of the convection–diffusion–reaction equation on distorted meshes using orthogonal subscales

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ABSTRACT

In this work a finite element approximation of the convection–diffusion–reaction equation on distorted meshes is described. The method is based on the decomposition of the unknown into its finite element component and a subgrid scale that needs to be approximated. The stability and accuracy of the final formulation depends on the approximation of the subscale problem, which is particularly challenging in the context of distorted meshes.

Our approximation, described in [2], takes into account the distortion of the mesh transforming the equation for the subgrid scale within each element to the shape-regular reference domain. Then, the expression for the subgrid scale arises from an approximate Fourier analysis, originally proposed in [1], and the identification of the wave number direction where instabilities are most likely to occur. The final outcome is an expression for the stabilization parameter that accounts for anisotropy and the dominance of either convection or reaction terms in the equation.

Another important ingredient of our approach is the choice of the subgrid scale space which is taken orthogonal to the finite element space (in the L^2 sense) [1]. This choice introduces a projection in the stabilization term that is essential to achieve optimal accuracy when the distortion of the mesh is severe.

References

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