

## A Conservative and Monotone Mixed–Hybridized Finite Element Approximation of Transport Problems in Heterogeneous Domains

M. Brera<sup>1</sup>, C. de Falco<sup>2</sup>, J. W. Jerome<sup>3</sup>, Y. Mori<sup>4</sup>, R. Sacco<sup>5</sup>

### ABSTRACT

In this lecture, we discuss the numerical approximation of transport phenomena occurring at material interfaces between physical subdomains with heterogeneous properties. The model in each subdomain consists of a partial differential equation with diffusive, convective and reactive terms, the coupling between each subdomain being realized through an interface transmission condition of Robin type. The numerical approximation of the problem in the two–dimensional case is carried out through a dual mixed–hybridized finite element method with numerical quadrature of the mass flux matrix. The resulting method is a conservative finite volume scheme over triangular grids, for which a discrete maximum principle is proved under the assumption that the mesh is of Delaunay type in the interior of the domain and of weakly acute type along the domain external boundary and internal interface. The stability, accuracy and robustness of the proposed method are validated on several numerical examples motivated by applications in Biology, Electrophysiology and Neuroelectronics.

<sup>1</sup>Master Degree in Electronic Engineering, Politecnico di Milano  
Piazza Leonardo da Vinci 32, 20133 Milano, Italy,  
[marko@fastwebnet.it](mailto:marko@fastwebnet.it)

<sup>2</sup>Dipartimento di Matematica “F. Brioschi”, Politecnico di Milano  
via Bonardi 9, 20133 Milano, Italy,  
[carlo.defalco@polimi.it](mailto:carlo.defalco@polimi.it)

<sup>3</sup>Department of Mathematics, Northwestern University  
2033 Sheridan Road, Evanston, IL 60208-2730, USA,  
[jwj@math.northwestern.edu](mailto:jwj@math.northwestern.edu)

<sup>4</sup>School of Mathematics, University of Minnesota,  
206 Church Street, Minneapolis, MN 55455, USA,  
[ymori@math.umn.edu](mailto:ymori@math.umn.edu)

<sup>5</sup>Dipartimento di Matematica “F. Brioschi”, Politecnico di Milano  
via Bonardi 9, 20133 Milano, Italy,  
[riccardo.sacco@polimi.it](mailto:riccardo.sacco@polimi.it)