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Higher order solution for a singularly perturbed diffusion-convection problem in two dimensions

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ABSTRACT

We consider the following problem

$$-\varepsilon\Delta u + \vec{b}(x,y)\cdot\nabla u + c(x,y)u = f \text{ in } \Omega; \quad u = 0 \text{ on } \Gamma \tag{1}$$

where $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$, $\Gamma_{\text{in}} = \Gamma_1 \cup \Gamma_2$, $\Gamma_{\text{out}} = \Gamma_3 \cup \Gamma_4$, and $\vec{b}^T = (b_1, b_2)$. We assume

$$\begin{cases} (b_1, b_2) > (\beta_1, \beta_2) > (0,0) \text{ in } \overline{\Omega}; \\ c + \frac{1}{2}\text{div} b \geq \gamma > 0 \end{cases} \tag{2}$$

Problem (1), a version of the Stokes problem is elliptic with a dominant convection term. Its limiting problem an hyperbolic one is given by

$$\vec{b}(x,y)\cdot\nabla v + c(x,y)v = f \text{ in } \Omega; \quad v = 0 \text{ on } \Gamma_{\text{in}} \qquad (3).$$

So, for $0 < \varepsilon << 1$ problem (1) exhibits a boundary layer phenomenon along the portion Γ_{out} of the boundary Γ. The current paper is intended to construct an approximation solution to u up to any higher order $\mathcal{O}(\varepsilon^q)$ where q is an arbitrary and prescribed non zero natural number. Our strategy relies on the computation of a corrector and on the use of the Hilbert sapce methods. We obtain also an exact localization of the boundary layer. This strategy provides also a precise localization of the boundary layer.

References

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