

## A priori estimates for a convection diffusion problem with a corner discontinuity

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### ABSTRACT

We consider the following boundary value problem in domain  $\Omega$ , the unit square.

$$\begin{aligned} Lu_\varepsilon &\equiv \varepsilon \Delta u_\varepsilon + p_1 \frac{\partial u_\varepsilon}{\partial x} + p_2 \frac{\partial u_\varepsilon}{\partial y} - qu_\varepsilon = f \text{ in } \Omega, \\ u_\varepsilon(x, 0) &= g_s(x), \quad u_\varepsilon(x, 1) = g_n(x), \quad x \in (0, 1) \\ u_\varepsilon(0, y) &= g_w(y), \quad u_\varepsilon(1, y) = g_e(y) \quad y \in (0, 1) \end{aligned} \tag{1}$$

where  $p_1, p_2$  and  $q$  are positive constants and  $0 < \varepsilon \leq 1$  and

$$f \in C^{2l, \alpha}(\bar{\Omega}), \quad g_s, g_n, g_w, g_e \in C^{2l, \alpha}([0, 1]) \tag{2}$$

for some non-negative integer  $l$  and  $\alpha \in (0, 1)$ .

We assume a discontinuity in the b.c.s at  $(0, 0)$ :  $g_s(0) \neq g_w(0)$ , but smoothness everywhere else. This problem was examined numerically in [1]; here we report on progress towards a priori estimates analogous to those obtained in [2, 3, 4].

## References

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