

The use of staggered grids as stabilization technique of the poro-elasticity problem and its efficient resolution by multigrid

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ABSTRACT

The classical quasi-static Biot model for soil consolidation, describes the time dependent interaction between the deformation of an elastic porous material and the fluid flow inside of it. This model can be formulated as a system of partial differential equations for the unknowns displacement and pressure. Physically, when a load is applied in a poro-elasticity medium, the pressure suddenly increases and a boundary layer appears in the early stages of the time-dependent process. In the case of using standard (no stabilized) discretization for the numerical solution of the problem, unphysical oscillations can occur in the approximation of pressure at the first time steps of the solution. After this phase, the solution shows a much smoother behavior. A staggered grid discretization approach has been presented in [2]. Pressure points in the staggered grid were located at the vertices of the cells while the displacement points were defined at the cell faces. This approach guarantees stable schemes, independently of discretization parameters, that are second order convergent in discrete energy norms. Finally, an efficient multigrid solver for the system of poroelasticity equations discretized on the staggered grid is developed. This multigrid method is based on distributive smoothers [1, 3] yielding in an efficient solver.

References

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