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Boundary layer problem: Navier-Stokes and Euler equations

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ABSTRACT

We consider the Navier-Stokes equations in a bounded domain $\Omega \subseteq \mathbf{R}^2$

$$\begin{aligned} \mathbf{v}_t + \operatorname{div}(\mathbf{v}\otimes\mathbf{v}) - \bigtriangledown p &= \mu\triangle\mathbf{v}, \qquad \mathbf{x}\in\Omega, \quad t>0, \\ \operatorname{div}\mathbf{v} &= 0, \\ \mathbf{v}(\mathbf{x},0) &= \mathbf{v}_0(\mathbf{x}), \qquad \mathbf{x}\in\Omega, \end{aligned}$$

admitting flows through the boundary $\partial\Omega$ of Ω

$$\begin{aligned} \mathbf{v}\cdot\mathsf{n} &= a(\mathbf{x},t), \qquad \mathbf{x}\in\partial\Omega, \quad t>0, \\ 2D(\mathbf{v})\mathsf{n}\cdot\mathsf{s} + \alpha(\mathbf{x},t)\ \mathbf{v}\cdot\mathsf{s} &= b(\mathbf{x},t). \end{aligned}$$

The last one are so-called Navier slip boundary conditions. Here $\mathbf{v}(\mathbf{x},t)$ - the velocity of the fluid; $p(\mathbf{x},t)$ - the pressure; $D(\mathbf{v}) := \frac{1}{2}[\nabla\mathbf{v}+(\nabla\mathbf{v})^T]$ - the rate-of-strain tensor of $\mathbf{v}$; (n,s) - the pair formed by the outside normal and tangent vectors to the boundary Γ of Ω.

The main result: *When* $\mu\to 0$ *we shown that the solutions* $\mathbf{v}_\mu$ *of the Navier - Stokes equations converge to the solution* $\mathbf{v}$ *of the Euler equations, satisfying the Navier slip boundary conditions on the part of the boundary* $\partial\Omega$, *where* $\mathbf{v}\cdot\mathsf{n} = a < 0$, *such that*

$$\mathbf{v}_\mu \to \mathbf{v} \qquad \textit{strongly in} \qquad L_\infty(0,T;W_p^1(\Omega)). \tag{1}$$

This result solved a so-called problem of boundary layers.

References

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