

Asymptotic behavior of a viscous fluid near a rough boundary

Casado-Díaz J.¹, Luna-Laynez M.¹, Suárez-Grau F.J.¹

ABSTRACT

The goal of this work is to study the asymptotic behavior of Navier-Stokes system in rugous domains by using an original adaptation of the unfolding method, which is closely related to the two-scale convergence method.

A well-accepted hypothesis in the fluid dynamics is that if the boundary Γ_ε of the physical domain is impermeable then the viscous fluid adheres completely to it. On the contrary, we assume that the fluid satisfies the slip condition given by Navier's law

$$u_\varepsilon \cdot n_\varepsilon = 0, \quad (Du_\varepsilon \cdot n_\varepsilon + \gamma u_\varepsilon)_{tan} = 0 \quad \text{on } \Gamma_\varepsilon,$$

where u_ε is the velocity, n_ε is the normal vector to Γ_ε , γ is the friction coefficient and the subscript tan denotes the tangential component.

Assuming Γ_ε periodic with period ε and amplitude δ_ε , we show that if $\delta_\varepsilon/\varepsilon^{3/2}$ tends to infinity then Navier's condition implies adherence condition in the limit. This justifies the use of the adherence condition in fluid dynamics. However, if $\delta_\varepsilon/\varepsilon^{3/2}$ tends to zero, the rugosity of the wall is worthless and the fluid behaves as if Γ_ε were a plane wall. Finally, if the limit of $\delta_\varepsilon/\varepsilon^{3/2}$ is in $(0, +\infty)$ (critical case), although the rugosity is not big enough to give an adherence condition, we obtain a new Navier's law with a larger friction coefficient. In general, this new coefficient is not a scalar but a matrix.

Moreover, we extend these results for PDE systems (not necessarily in fluid mechanics) with rugose boundaries, not necessarily periodic.

References

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¹Dpto. de Ecuaciones Diferenciales y Anlisis Numrico
Universidad de Sevilla
Dpto. E.D.A.N. Facultad de Matemáticas 41012 SEVILLA
jcasadod@us.es, mllaynez@us.es, fjsgrau@us.es